

$\Upsilon(1S) \rightarrow B_c\pi, B_cK$ decays with perturbative QCD approach

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Abstract

With the potential prospects of the $\Upsilon(1S)$ at high-luminosity dedicated heavy-flavor factories, the bottom-changing $\Upsilon(1S) \rightarrow B_c\pi, B_cK$ weak decays are studied with the pQCD approach. It is found that branching ratio for the color-favored and CKM-favored $\Upsilon(1S) \rightarrow B_c\pi$ decay can reach up to $\mathcal{O}(10^{-11})$. So the $\Upsilon(1S) \rightarrow B_c\pi$ decay might be measured promisingly by the future experiments.

I. INTRODUCTION

The $\Upsilon(1S)$ particle is the ground vector bottomonia (bound states of $b\bar{b}$) with well established quantum number of $I^G J^{PC} = 0^{-1}--$ [1]. The mass of the $\Upsilon(1S)$ particle, $m_{\Upsilon(1S)} = 9460.30 \pm 0.26$ MeV [1], is less than the kinematic $B\bar{B}$ threshold. The $\Upsilon(1S)$ particle, in a close analogy with J/ψ , decay primarily through the annihilation of the $b\bar{b}$ pairs into three gluons, followed by evolution of gluons into hadrons, glueballs, hybrid, multiquark and other exotic states. The hadronic $\Upsilon(1S)$ decay offers an ideal place to reap the properties of the invisible gluons and of the quark-gluon coupling [2]. It is well known that strong decay of the $\Upsilon(1S)$ particle is suppressed by the phenomenological Okubo-Zweig-Iizuka (OZI) rules [3–5], so electromagnetic and radiative transitions become competitive. Besides, the $\Upsilon(1S)$ particle can also decay via the weak interactions within the standard model, although the branching ratio is very small, about $2/\tau_B \Gamma_{\Upsilon(1S)} \sim \mathcal{O}(10^{-8})$ [1], where τ_B and $\Gamma_{\Upsilon(1S)}$ are the lifetime of the $B_{u,d,s}$ meson and the decay width of the $\Upsilon(1S)$ particle, respectively. In this paper, we will estimate the branching ratios for the bottom-changing nonleptonic $\Upsilon(1S) \rightarrow B_c \pi$, $B_c K$ weak decays with perturbative QCD (pQCD) approach [6–8]. The motivation is listed as follows.

From the experimental point of view, (1) over 10^8 $\Upsilon(1S)$ samples have been accumulated at Belle [9]. Many more upsilons could be collected with great precision at the forthcoming SuperKEKB and the running upgraded LHC. The abundant $\Upsilon(1S)$ samples provide a golden opportunity to search for the $\Upsilon(1S)$ weak decays which in some cases might be detectable. Theoretical studies on the $\Upsilon(1S)$ weak decays are just necessary to offer a ready reference. (2) For the two-body $\Upsilon(1S) \rightarrow B_c \pi$, $B_c K$ decays, final states with opposite charges have definite energies and momenta in the rest frame of the $\Upsilon(1S)$ particle. Besides, identification of a single explicitly flavored B_c meson is free from inefficiently double tagging above the $B\bar{B}$ threshold [10], and can provide a conclusive evidence of the $\Upsilon(1S)$ weak decay. Of course, small branching ratios make the observation of the $\Upsilon(1S)$ weak decays extremely challenging, and evidences of an abnormally large production rate of single B_c mesons in the $\Upsilon(1S)$ decay might be a hint of new physics [10].

From the theoretical point of view, the bottom-changing upyon weak decays permit one to cross check parameters obtained from B meson decay. The color-favored $\Upsilon(1S) \rightarrow B_c \pi$, $B_c K$ decays have been estimated with the naive factorization (NF) approximation

in previous works [10–12]. An obvious deficiency of NF approximation is the absence of strong phases and the renormalization scale from hadronic matrix elements (HME). Recently, several attractive methods have been developed to evaluate HME, such as pQCD [6–8], the QCD factorization (QCDF) [13–15] and soft and collinear effective theory [16–19], which could give reasonable explanation for many measurements on $B_{u,d}$ hadronic decays. The $\Upsilon(1S) \rightarrow B_c\pi$, B_cK decays are calculated at the next-to-leading (NLO) order with the QCDF approach [20]. In this paper, the $\Upsilon(1S) \rightarrow B_c\pi$, B_cK weak decays will be evaluated with the pQCD approach to check the consistency of prediction on branching ratios among different models.

This paper is organized as follows. In section II, we present the theoretical framework and the amplitudes for the $\Upsilon(1S) \rightarrow B_c\pi$, B_cK decays with the pQCD approach. Section III is devoted to numerical results and discussion. The last section is our summary.

II. THEORETICAL FRAMEWORK

A. The effective Hamiltonian

The effective Hamiltonian responsible for the $\Upsilon(1S) \rightarrow B_c\pi$, B_cK decays is [21]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q=d,s} V_{cb}V_{uq}^* \left\{ C_1(\mu) Q_1(\mu) + C_2(\mu) Q_2(\mu) \right\} + \text{H.c.}, \quad (1)$$

where $G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2}$ [1] is the Fermi coupling constant; the Cabibbo-Kabayashi-Maskawa (CKM) factors are expanded as a power series in the Wolfenstein parameter $\lambda \sim 0.2$ [1],

$$V_{cb}V_{ud}^* = A\lambda^2 - \frac{1}{2}A\lambda^4 - \frac{1}{8}A\lambda^6 + \mathcal{O}(\lambda^8), \quad (2)$$

$$V_{cb}V_{us}^* = A\lambda^3 + \mathcal{O}(\lambda^8). \quad (3)$$

The Wilson coefficients $C_{1,2}(\mu)$ summarize the physical contributions above scales of μ , and have properly been calculated to the NLO order with the renormalization group improved perturbation theory. The local operators are defined as follows.

$$Q_1 = [\bar{c}_\alpha \gamma_\mu (1 - \gamma_5) b_\alpha] [\bar{q}_\beta \gamma^\mu (1 - \gamma_5) u_\beta], \quad (4)$$

$$Q_2 = [\bar{c}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta] [\bar{q}_\beta \gamma^\mu (1 - \gamma_5) u_\alpha], \quad (5)$$

where α and β are color indices and the sum over repeated indices is understood.

From the effective Hamiltonian Eq.(1), it can be easily seen that only tree operators with coupling strength proportional to the CKM element V_{cb} contribute to the $\Upsilon(1S) \rightarrow B_c\pi$, B_cK decays, and there is no pollution from penguin and annihilation contributions.

B. Hadronic matrix elements

To obtain the decay amplitudes, the remaining works are how to calculate accurately hadronic matrix elements of local operators. Using the Lepage-Brodsky approach for exclusive processes [22], HME could be expressed as the convolution of hard scattering subamplitudes containing perturbative contributions with the universal wave functions reflecting the nonperturbative contributions. Sometimes the high-order corrections to HME produce collinear and/or soft logarithms based on collinear factorization approximation, for example, the spectator scattering amplitudes within the QCDF framework [15]. The pQCD approach advocates that [6–8] this inconsistent treatment on HME could be smeared by retaining the transverse momentum of quarks and introducing the Sudakov factor. The decay amplitudes could be factorized into three parts: the “harder” effects incorporated into the Wilson coefficients C_i , the process-dependent heavy quark decay subamplitudes H , and the universal wave functions Φ ; and are written as

$$\int dx db C_i(t) H(t, x, b) \Phi(x, b) e^{-S}, \quad (6)$$

where t is a typical scale, x is the longitudinal momentum fraction of the valence quark, b is the conjugate variable of the transverse momentum, and e^{-S} is the Sudakov factor.

C. Kinematic variables

The light cone kinematic variables in the $\Upsilon(1S)$ rest frame are defined as follows.

$$p_\Upsilon = p_1 = \frac{m_1}{\sqrt{2}}(1, 1, 0), \quad (7)$$

$$p_{B_c} = p_2 = (p_2^+, p_2^-, 0), \quad (8)$$

$$p_{\pi(K)} = p_3 = (p_3^-, p_3^+, 0), \quad (9)$$

$$k_i = x_i p_i + (0, 0, \vec{k}_{i\perp}), \quad (10)$$

$$\epsilon_{\Upsilon}^{\parallel} = \frac{1}{\sqrt{2}}(1, -1, 0), \quad (11)$$

$$n_+ = (1, 0, 0), \quad n_- = (0, 1, 0), \quad (12)$$

where x_i and $\vec{k}_{i\perp}$ are the longitudinal momentum fraction and transverse momentum of the light valence quark, respectively; $\epsilon_{\Upsilon}^{\parallel}$ is the longitudinal polarization vector of the $\Upsilon(1S)$ particle; n_+ and n_- are the positive and negative null vectors, respectively. The notation of momentum is displayed in Fig.1(a).

The relations among these kinematic variables are

$$p_i^{\pm} = \frac{E_i \pm p}{\sqrt{2}}, \quad (13)$$

$$p = \frac{\sqrt{\lambda(m_1^2, m_2^2, m_3^2)}}{2m_1}, \quad (14)$$

$$s = 2p_2 \cdot p_3 = m_1^2 - m_2^2 - m_3^2, \quad (15)$$

$$t = 2p_1 \cdot p_2 = m_1^2 + m_2^2 - m_3^2, \quad (16)$$

$$u = 2p_1 \cdot p_3 = m_1^2 - m_2^2 + m_3^2, \quad (17)$$

$$t + u - s = m_1^2 + m_2^2 + m_3^2, \quad (18)$$

$$s t + s u - u t = \lambda(m_1^2, m_2^2, m_3^2), \quad (19)$$

$$\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc, \quad (20)$$

where p is the common momentum of final states; m_1 , m_2 and m_3 denote the masses of the $\Upsilon(1S)$, B_c and $\pi(K)$ mesons, respectively.

D. Wave functions

With the notation in [23–25], the explicit definitions of matrix elements of diquark operators sandwiched between vacuum and the longitudinally polarized $\Upsilon(1S)$, the double-heavy pseudoscalar B_c , the light pseudoscalar P ($= \pi, K$) are

$$\langle 0 | b_i(z) \bar{b}_j(0) | \Upsilon(p_1, \epsilon_{\parallel}) \rangle = \frac{1}{4} f_{\Upsilon} \int dx_1 e^{-ix_1 p_1 \cdot z} \left\{ \not{\epsilon}_{\parallel} \left[m_1 \phi_{\Upsilon}^v(x_1) - \not{p}_1 \phi_{\Upsilon}^t(x_1) \right] \right\}_{ji}, \quad (21)$$

$$\langle B_c^+(p_2) | \bar{c}_i(z) b_j(0) | 0 \rangle = \frac{i}{4} f_{B_c} \int dx_2 e^{ix_2 p_2 \cdot z} \left\{ \gamma_5 \left[\not{p}_2 + m_2 \right] \phi_{B_c}(x_2) \right\}_{ji}, \quad (22)$$

$$\begin{aligned} & \langle P(p_3) | u_i(z) \bar{q}_j(0) | 0 \rangle \\ &= \frac{i}{4} f_P \int dx_3 e^{i x_3 p_3 \cdot z} \left\{ \gamma_5 \left[\not{p}_3 \phi_P^a(x_3) + \mu_P \phi_P^p(x_3) - \mu_P (\not{n}_- \not{n}_+ - 1) \phi_P^t(x_3) \right] \right\}_{ji}, \end{aligned} \quad (23)$$

where f_Υ , f_{B_c} , f_P are decay constants, $\mu_P = m_3^2/(m_u + m_q)$ and $q = d(s)$ for $\pi(K)$ meson.

The leading twist distribution amplitudes of light pseudoscalar π , K mesons are defined in terms of Gegenbauer polynomials [25]:

$$\phi_P^a(x) = 6 x \bar{x} \left\{ 1 + \sum_{n=1}^{\infty} a_n^P C_n^{3/2}(x - \bar{x}) \right\}, \quad (24)$$

where $\bar{x} = 1 - x$; a_n^P and $C_n^{3/2}(z)$ are Gegenbauer moment and polynomials, respectively; $a_i^\pi = 0$ for $i = 1, 3, 5, \dots$ due to the G -parity invariance of the pion distribution amplitudes.

Because of $m_{\Upsilon(1S)} \simeq 2m_b$ and $m_{B_c} \simeq m_b + m_c$, both $\Upsilon(1S)$ and B_c systems are nearly nonrelativistic, which can play the same role in understanding hadronic dynamics as the positronium and hydrogen atom in understanding the atomic physics [26]. Nonrelativistic quantum chromodynamics (NRQCD) [27–29] and Schrödinger equation can be used to describe their spectrum and thus one can learn about the interquark binding forces responsible for these states [2]. The eigenfunction of the time-independent Schrödinger equation with scalar harmonic oscillator potential corresponding to the quanta $nL = 1S$ is written as

$$\phi(\vec{k}) \sim e^{-\vec{k}^2/2\beta^2}, \quad (25)$$

where the parameter β determines the average transverse momentum, i.e., $\langle 1S | \vec{k}_\perp^2 | 1S \rangle = \beta^2$. According to the NRQCD power counting rules [27], the characteristic magnitude of the momentum is order of Mv , where M is the mass of the heavy quark with typical velocity $v \sim \alpha_s(M)$. Thus we will take $\beta = M\alpha_s(M)$ in our calculation. Employing the substitution ansatz [30],

$$\vec{k}^2 \rightarrow \frac{1}{4} \sum_i \frac{\vec{k}_{i\perp}^2 + m_{q_i}^2}{x_i}, \quad (26)$$

where x_i , $\vec{k}_{i\perp}$, m_{q_i} are the longitudinal momentum fraction, transverse momentum, mass of the light valence quark, respectively, with the relations $\sum x_i = 1$ and $\sum \vec{k}_{i\perp} = 0$. Integrating out $\vec{k}_{i\perp}$ and combining with their asymptotic forms, one can obtain

$$\phi_{B_c}(x) = A x \bar{x} \exp \left\{ - \frac{\bar{x} m_c^2 + x m_b^2}{8 \beta_2^2 x \bar{x}} \right\}, \quad (27)$$

$$\phi_\Upsilon^v(x) = B x \bar{x} \exp \left\{ - \frac{m_b^2}{8 \beta_1^2 x \bar{x}} \right\}, \quad (28)$$

$$\phi_{\Upsilon}^t(x) = C (x - \bar{x})^2 \exp\left\{-\frac{m_b^2}{8\beta_1^2 x \bar{x}}\right\}, \quad (29)$$

where $\beta_i = \xi_i \alpha_s(\xi_i)$ with $\xi_i = m_i/2$; parameters A, B, C are the normalization coefficients satisfying the conditions

$$\int_0^1 dx \phi_{B_c}(x) = 1, \quad \int_0^1 dx \phi_{\Upsilon}^{v,t}(x) = 1. \quad (30)$$

E. Decay amplitudes

The Feynman diagrams for $\Upsilon(1S) \rightarrow B_c \pi$ decay are shown in Fig.1, where (a) and (b) are factorizable topology; (c) and (d) are nonfactorizable topology.

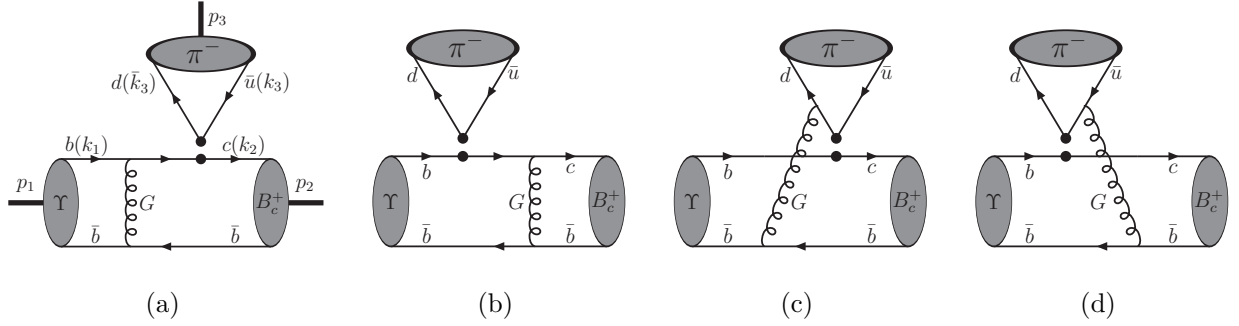


FIG. 1: Feynman diagrams for the $\Upsilon \rightarrow B_c \pi$ decay with the pQCD approach.

With the master formula Eq.(6), the decay amplitudes of $\Upsilon(1S) \rightarrow B_c P$ ($P = \pi$ and K) decay can be written as

$$\mathcal{A}(\Upsilon(1S) \rightarrow B_c P) = \sqrt{2} G_F \frac{\pi C_F}{N} V_{cb} V_{uq}^* m_{\Upsilon}^3 p f_{\Upsilon} f_{B_c} f_P \sum_{i=a,b,c,d} \mathcal{A}_{\text{Fig.1(i)}}, \quad (31)$$

where $C_F = 4/3$ and the color number $N = 3$.

The explicit expressions of $\mathcal{A}_{\text{Fig.1(i)}}$ are

$$\begin{aligned} \mathcal{A}_{\text{Fig.1(a)}} &= \int_0^1 dx_1 \int_0^\infty b_1 db_1 \int_0^1 dx_2 \int_0^\infty b_2 db_2 \alpha_s(t_a) a_1(t_a) E_a(t_a) \\ &\quad \times \phi_{\Upsilon}^v(x_1) \phi_{B_c}(x_2) H_a(x_1, x_2, b_1, b_2) \left\{ x_2 + r_2 r_b + r_3^2 \bar{x}_2 \right\}, \end{aligned} \quad (32)$$

$$\begin{aligned} \mathcal{A}_{\text{Fig.1(b)}} &= \int_0^1 dx_1 \int_0^\infty b_1 db_1 \int_0^1 dx_2 \int_0^\infty b_2 db_2 \alpha_s(t_b) a_1(t_b) E_b(t_b) \\ &\quad \times \phi_{B_c}(x_2) H_b(x_1, x_2, b_2, b_1) \left\{ \phi_{\Upsilon}^v(x_1) \left[2r_2 r_c - r_2^2 x_1 - r_3^2 \bar{x}_1 \right] \right. \\ &\quad \left. + \phi_{\Upsilon}^t(x_1) \left[2r_2 x_1 - r_c \right] \right\}, \end{aligned} \quad (33)$$

$$\begin{aligned}
\mathcal{A}_{\text{Fig.1(c)}} &= \int_0^1 dx_1 \int_0^\infty db_1 \int_0^1 dx_2 \int_0^\infty b_2 db_2 \int_0^1 dx_3 \int_0^\infty b_3 db_3 \delta(b_1 - b_2) \\
&\times \alpha_s(t_c) \frac{C_2(t_c)}{N} E_c(t_c) \phi_{B_c}(x_2) \phi_P^a(x_3) H_c(x_1, x_2, x_3, b_2, b_3) \\
&\times \left\{ \phi_Y^v(x_1) \left[\frac{t(x_1 - x_3)}{m_1^2} + 2r_2^2(x_3 - x_2) \right] + \phi_Y^t(x_1) r_2(x_2 - x_1) \right\}, \tag{34}
\end{aligned}$$

$$\begin{aligned}
\mathcal{A}_{\text{Fig.1(d)}} &= \int_0^1 dx_1 \int_0^\infty db_1 \int_0^1 dx_2 \int_0^\infty b_2 db_2 \int_0^1 dx_3 \int_0^\infty b_3 db_3 \delta(b_1 - b_2) \\
&\times \alpha_s(t_d) \frac{C_2(t_d)}{N} E_d(t_d) \phi_{B_c}(x_2) \phi_P^a(x_3) H_d(x_1, x_2, x_3, b_2, b_3) \\
&\times \left\{ \phi_Y^v(x_1) \frac{s(\bar{x}_2 - x_3)}{m_1^2} + \phi_Y^t(x_1) r_2(x_2 - x_1) \right\}, \tag{35}
\end{aligned}$$

where α_s is the QCD coupling; $a_1 = C_1 + C_2/N$; $C_{1,2}$ is the Wilson coefficients; $r_i = m_i/m_1$. It can be easily seen that the nonfactorizable contributions $\mathcal{A}_{\text{Fig.1(c,d)}}$ are color-suppressed with respect to the factorizable contributions $\mathcal{A}_{\text{Fig.1(a,b)}}$.

The typical scales t_i and the Sudakov factor E_i are defined as

$$t_{a(b)} = \max(\sqrt{-\alpha_g}, \sqrt{-\beta_{a(b)}}, 1/b_1, 1/b_2), \tag{36}$$

$$t_{c(d)} = \max(\sqrt{-\alpha_g}, \sqrt{|\beta_{c(d)}|}, 1/b_1, 1/b_2, 1/b_3), \tag{37}$$

$$E_{a(b)}(t) = \exp\{-S_{B_c}(t)\}, \tag{38}$$

$$E_{c(d)}(t) = \exp\{-S_{B_c}(t) - S_P(t)\}, \tag{39}$$

$$\alpha_g = \bar{x}_1^2 m_1^2 + \bar{x}_2^2 m_2^2 - \bar{x}_1 \bar{x}_2 t, \tag{40}$$

$$\beta_a = m_1^2 - m_b^2 + \bar{x}_2^2 m_2^2 - \bar{x}_2 t, \tag{41}$$

$$\beta_b = m_2^2 - m_c^2 + \bar{x}_1^2 m_1^2 - \bar{x}_1 t, \tag{42}$$

$$\beta_c = x_1^2 m_1^2 + x_2^2 m_2^2 + x_3^2 m_3^2 - x_1 x_2 t - x_1 x_3 u + x_2 x_3 s, \tag{43}$$

$$\beta_d = \bar{x}_1^2 m_1^2 + \bar{x}_2^2 m_2^2 + x_3^2 m_3^2 - \bar{x}_1 \bar{x}_2 t - \bar{x}_1 x_3 u + \bar{x}_2 x_3 s, \tag{44}$$

$$S_{B_c}(t) = s(x_2, p_2^+, 1/b_2) + 2 \int_{1/b_2}^t \frac{d\mu}{\mu} \gamma_q, \tag{45}$$

$$S_P(t) = s(x_3, p_3^+, 1/b_3) + s(\bar{x}_3, p_3^+, 1/b_3) + 2 \int_{1/b_3}^t \frac{d\mu}{\mu} \gamma_q, \tag{46}$$

where α_g and β_i are the virtuality of the internal gluon and quark, respectively; $\gamma_q = -\alpha_s/\pi$ is the quark anomalous dimension; the expression of $s(x, Q, 1/b)$ can be found in the appendix of Ref.[6].

The scattering functions H_i in the subamplitudes $\mathcal{A}_{\text{Fig.1(i)}}$ are defined as

$$H_{a(b)}(x_1, x_2, b_i, b_j) = K_0(\sqrt{-\alpha_g b_i}) \left\{ \theta(b_i - b_j) K_0(\sqrt{-\beta b_i}) I_0(\sqrt{-\beta b_j}) + (b_i \leftrightarrow b_j) \right\}, \quad (47)$$

$$\begin{aligned} H_{c(d)}(x_1, x_2, x_3, b_2, b_3) &= \left\{ \theta(-\beta) K_0(\sqrt{-\beta b_3}) + \frac{\pi}{2} \theta(\beta) [i J_0(\sqrt{\beta b_3}) - Y_0(\sqrt{\beta b_3})] \right\} \\ &\times \left\{ \theta(b_2 - b_3) K_0(\sqrt{-\alpha_g b_2}) I_0(\sqrt{-\alpha_g b_3}) + (b_2 \leftrightarrow b_3) \right\}, \end{aligned} \quad (48)$$

where J_0 and Y_0 (I_0 and K_0) are the (modified) Bessel function of the first and second kind, respectively.

III. NUMERICAL RESULTS AND DISCUSSION

In the rest frame of the $\Upsilon(1S)$ particle, branching ratio for the $\Upsilon(1S) \rightarrow B_c P$ weak decays can be written as

$$\mathcal{B}r(\Upsilon(1S) \rightarrow B_c P) = \frac{1}{12\pi} \frac{p}{m_\Upsilon^2 \Gamma_\Upsilon} |\mathcal{A}(\Upsilon(1S) \rightarrow B_c P)|^2, \quad (49)$$

where the decay width $\Gamma_\Upsilon = 54.02 \pm 1.25$ keV [1].

The values of input parameters are listed as follows.

- (1) Wolfenstein parameters [1]: $A = 0.814_{-0.024}^{+0.023}$ and $\lambda = 0.22537 \pm 0.00061$.
- (2) Masses of mesons [1]: $m_{B_c} = 6275.6 \pm 1.1$ MeV and $m_{\Upsilon(1S)} = 9460.30 \pm 0.26$ MeV.
- (3) Masses of quarks [1]: $m_c = 1.67 \pm 0.07$ GeV and $m_b = 4.78 \pm 0.06$ GeV.
- (4) Gegenbauer moments at the scale of $\mu = 1$ GeV: $a_2^\pi = 0.17 \pm 0.08$ and $a_4^\pi = 0.06 \pm 0.10$ [31] for twist-2 pion distribution amplitudes, $a_1^K = 0.06 \pm 0.03$ and $a_2^K = 0.25 \pm 0.15$ [25] for twist-2 kaon distribution amplitudes.
- (5) Decay constants: $f_\pi = 130.41 \pm 0.20$ MeV [1], $f_K = 156.2 \pm 0.7$ MeV [1], $f_{B_c} = 489 \pm 5$ MeV [32]. As for the decay constant f_Υ , one can use the definition of decay constant f_V for vector meson V with mass m_V and polarization vector ϵ_V ,

$$\langle 0 | \bar{\psi} \gamma^\mu \psi | V \rangle = f_V m_V \epsilon_V^\mu. \quad (50)$$

The decay constant f_V is related to the experimentally measurable leptonic branching ratio:

$$\Gamma(V \rightarrow \ell^+ \ell^-) = \frac{4\pi}{3} \alpha_{\text{QED}}^2 Q_q^2 \frac{f_V^2}{m_V} \sqrt{1 - 2 \frac{m_\ell^2}{m_V^2}} \left\{ 1 + 2 \frac{m_\ell^2}{m_V^2} \right\}, \quad (51)$$

where α_{QED} is the fine-structure constant, m_ℓ is the lepton mass, Q_q is the electric charge of the quark in the unit of $|e|$, and $Q_b = -1/3$ for the bottom quark. The experimental measurements on leptonic $\Upsilon(1S)$ decays give the weighted average decay constant $f_\Upsilon = (676.4 \pm 10.7)$ MeV (see Table.I).

TABLE I: Branching ratios for leptonic $\Upsilon(1S)$ decays and decay constants f_Υ , where the last column is the weighted average, and errors come from mass, width and branching ratios.

decay mode	branching ratio	decay constant	
$\Upsilon(1S) \rightarrow e^+e^-$	$(2.38 \pm 0.11)\%$	(664.2 ± 23.1) MeV	
$\Upsilon(1S) \rightarrow \mu^+\mu^-$	$(2.48 \pm 0.05)\%$	(677.9 ± 14.7) MeV	(676.4 ± 10.7) MeV
$\Upsilon(1S) \rightarrow \tau^+\tau^-$	$(2.60 \pm 0.10)\%$	(683.3 ± 21.1) MeV	

TABLE II: Branching ratios for the $\Upsilon(1S) \rightarrow B_c\pi$, B_cK decays, where the results of Refs. [11, 12, 20] are calculated with the coefficient $a_1 = 1.05$. The uncertainties of the last column come from the CKM parameters, the renormalization scale $\mu = (1 \pm 0.1)t_i$, masses of b and c quarks, hadronic parameters (decay constants and Gegenbauer moments), respectively.

	Ref. [11]	Ref. [12]	Ref. [20]	this work
$10^{11} \times \mathcal{B}r(\Upsilon(1S) \rightarrow B_c\pi)$	6.91	2.8	5.03	$7.04^{+0.48+0.80+0.83+0.47}_{-0.46-0.52-0.98-0.44}$
$10^{12} \times \mathcal{B}r(\Upsilon(1S) \rightarrow B_cK)$	5.03	2.3	3.73	$5.41^{+0.40+0.63+0.64+0.39}_{-0.38-0.41-0.74-0.37}$

If not specified explicitly, we will take their central values as the default inputs. Our numerical results on the CP -averaged branching ratios for the $\Upsilon(1S) \rightarrow B_c\pi$, B_cK decays are displayed in Table II, where the uncertainties come from the CKM parameters, the renormalization scale $\mu = (1 \pm 0.1)t_i$, masses of b and c quarks, hadronic parameters (decay constants and Gegenbauer moments), respectively. The following are some comments.

(1) The pQCD's results on branching ratios for the $\Upsilon(1S) \rightarrow B_c\pi$, B_cK decays have the same magnitude as those of Refs. [11, 12, 20]. The estimation of Refs. [11, 12] is based on the NF approximation, where nonfactorizable corrections to HME are not considered, and the form factors for the transition between $\Upsilon(1S)$ and B_c mesons are calculated with the heavy quark effective theory in Ref. [11] and the Wirbel-Stech-Bauer [33] model in Ref. [12], respectively. The coefficient a_1 containing the NLO nonfactorizable contributions to

HME are used in Ref. [20] within the QCDF framework, where the form factors are written as the overlap integrals of nonrelativistic wave functions for $\Upsilon(1S)$ and B_c mesons based on the Wirbel-Stech-Bauer model. Compared with the NF and QCDF approach, there are more contributions from the nonfactorizable decay amplitudes $\mathcal{A}_{\text{Fig.1(c,d)}}$ with the pQCD approach. This may be why the pQCD's results are slightly larger than previous ones.

(2) Because the CKM factors $|V_{cb}V_{us}^*| < |V_{cb}V_{ud}^*|$, there is a relation between branching ratios, $\mathcal{B}r(\Upsilon(1S) \rightarrow B_c \pi) > \mathcal{B}r(\Upsilon(1S) \rightarrow B_c K)$.

(3) Branching ratio for the $\Upsilon(1S) \rightarrow B_c \pi$ decay can reach up to 10^{-11} . So the $\Upsilon(1S) \rightarrow B_c \pi$ decay should be sought for with high priority and first observed at the running LHC and forthcoming SuperKEKB. For example, the $\Upsilon(1S)$ production cross section in p-Pb collision can reach up to a few μb with the LHCb [34] and ALICE [35] detectors at LHC. Over 10^{11} $\Upsilon(1S)$ particles per 100 fb^{-1} data collected at LHCb and ALICE are in principle available, corresponding to a few tens of $\Upsilon(1S) \rightarrow B_c \pi$ events.

(4) There are many uncertainties on our results. Other factors, such as the contributions of higher order corrections to HME, relativistic effects and so on, which are not considered here, deserve the dedicated study. Our results just provide an order of magnitude estimation.

IV. SUMMARY

The $\Upsilon(1S)$ weak decay is legal within the standard model, although branching ratio is tiny compared with the strong and electromagnetic decays. With the potential prospects of the $\Upsilon(1S)$ at high-luminosity dedicated heavy-flavor factories, the bottom-changing $\Upsilon(1S) \rightarrow B_c \pi$, $B_c K$ weak decays are studied with the pQCD approach. It is found that with the nonrelativistic wave functions for $\Upsilon(1S)$ and B_c mesons, branching ratios for the $\Upsilon(1S) \rightarrow B_c \pi$, $B_c K$ decays have the same order as previous works, and $\mathcal{B}r(\Upsilon(1S) \rightarrow B_c \pi) \gtrsim 10^{-11}$. The color-favored and CKM-favored $\Upsilon(1S) \rightarrow B_c \pi$ decay might be detectable in future experiments.

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